Distilling The Unit

# Prereq and Basics

## Transistor Equivalent Circuit

## Simple Voltage Amplifier

## Thevenin's Theorem

"Any combination of voltage sources and resistors with two terminals can be replaced by a single voltage source and a single series resistor."  
We can find this equivalent by:

1. Calculate the **open circuit voltage** and the **short-circuit current** of Circuit A.
2. Then calculate Thevenin resistance as:   
   (Can be done with and only

## Amplifiers

Signal gain can be achieved and expressed through voltage, current, or power.

Gain is a ratio, expressed as dimensionless, or V/V, A/A etc. Engineers often express gain logarithmically.

# Diodes

* Fabricated using a semiconductor with a P-N Junction
* Applications:  
  - Power conversion  
  - Over voltage protection  
  - Detectors  
  - Signal Generation

## Shockley Equation

*Shockley Equation (Forward bias)*

**Where:**

* Saturation current. Its value depends on the area of the diode and the temp. Doubling for each increase for silicon diodes. A at 300K
* Emission coefficient (ideality factor)
* Thermal voltage
  + Temperature in K = 273 + Temp in C
* Joules/Kelvin (**Boltzmann's Constant**)
* Coulomb (Charge of an electron)

There is also an approximate Shockley (for forward bias more than several tenths of a volt)

## Constant Voltage Drop Model

* Only accounts for the forward turn on voltage
* Does not account for the slope or reverse breakdown.
* For silicon diodes, typical

## Linear Model (Not Accurate)

We can also model our diode using a simple linear equation. Although this is not a highly accurate model of a diode.

From this we derive:

We can improve this using piecewise functions to generalize non-linearity

*This is our graphical model. Good for load-line analysis*

## Analysis

### Assumed States

*In a circuit with a number of diodes n*

* Assume a state for each diode (ON/OFF)(Closed/Open)
* n diodes will require possible combinations
* **Calculate currents of ON diodes and voltages of OFF diodes**
* Check to see of calculations are consistent with assumptions
* If yes, assumption works. *If no, return to step 1 and make a new assumption*.

### Load Line Analysis

There's no straightforward way to solve the Shockley Equation with Kirchhoff's Voltage Law ()  
We use load-line analysis graphically. Since is upside down here, what this means is that when there is no reverse bias, . When full reverse bias there is no current.

*We also need these to find the operating point:*  
 (constant drop)

#### Load Line Analysis of Complex Circuits

### Small Signal Analysis (AC Signals)

* We applied *DC voltages (or currents) to the diodes and found the operating points (Q point)*
* Q stands for **quiescent** i.e. quiet or no-signal point
* \*If the Q point is known (using the nonlinear diode characteristic), the linear AC analysis method can be applied to find the response to small **(AC) signals.**

## Zener Diode

Zener diodes are designed to work at the reverse breakdown region. These diodes use this as expected behaviour.

*Commonly used for voltage regulator circuits - constant output from variable supply.*  
5V change in the supply voltage results in only a 0.5V change the regulated output voltage.

## Rectifiers and Regulation

### Half Wave

*We can use the 0.7V drop model or ideal diode model without losing much accuracy because voltage levels >> 0.7V*  
We can add a capacitor to smooth the output

**Where:**  
 Average load current  
 Period of AC voltage  
 Ripple voltage

*Chose diode to withstand Peak Inverse Voltage*  
  
PIV needs to be less than the breakdown voltage of the diode.

# BJTs

The Arrow shows the current of the controlling pn-Junction

* Arrow from Emitter = PNP
* Arrow TO Emitter = NPN

## Modes of Operation

*Operating mode depends on DC biasing or large signal voltages and currents:*

* **Active** mode - Used for AC amplification
* **Cutoff** and *saturation* modes - used for switching
* BJT is not symmetrical - inverse region is not normally used.

### Active Region

Using Kirchhoff:

We can define:

Which Implies:

Where:  
 = Factor of proportionality  
 Emitter injection efficiency

Typical values of these include:

*Some derivation I guess:*

![](data:image/png;base64;base64,)

NOTE

Base-emitter voltage controls the collector current

*Emitter current is equal to the total current:*

Using Kirchhoff's Laws and our diode equation:  
Where:  
 Constant (Saturation Current)  
 Constant (Thermal Voltage)

We can determine:  
*Base current is much smaller that collector current*

Emitter current is the total current:

### Saturation Region

( at saturation) <=

## Load Line Analysis

This gives us our operating point that allows that irrespective of AC swing the diode remains in the active region.

## Constant Voltage Drop Approximation

In the active region we can use a constant voltage (base side) and constant current (collector side) approximation

## Temperature and Thermal Voltage

![](data:image/png;base64;base64,)

Note

At a **constant current** the voltage decreases by approximately **2mV** for every 1 Degree C increase in temperature. (Same as PN Diode)

Thermal voltage at room temp is 25mV

## BJT Amplifier and DC Biasing

We can use to **Bias the transistor into active mode**

**Small Signal Voltage Gain:**

* Gain is negative **inverting amp**
* Gain **depends on bias point (Q point)**
* *Only valid for small signal*

**Output current and output voltage**

**Output current and output voltage after capacitors**  
*if*

### Setting Bias Point

*Input circuit*  
We bias the circuit assuming 0 small signal voltage

## Transconductance

Transconductance is the ratio of the current flowing through the output and the voltage arising in the input of electrical circuit/devices. Transconductance is calculated using the equation.

**Total = DC + Signal**

Where is our signal

## Small Signal AC

### Hybrid Pi Model

### T Model

# MOSFETS

* **Metal Oxide Semiconductor Field Effect Transistor**
  + **Metal** - Used for the contacts
  + **Oxide** - Unlike BJTs, in a MOSFET the gate is separated from the rest of the device with a thin insulating layer
  + **Semiconductor** - Commonly fabricated on silicon (Although SiGe can be used)
  + **Field Effect** - Applying a voltage between the gate and body terminals creates an electric field which penetrates the oxide and creates an inversion layer at the semiconductor interface.

## Device Operation

* Channel is induced when **gate source voltage exceeds the threshold voltage** (Given, around 1V)
* Additional voltage beyond the threshold point is the **overdrive**
* Drain current has a square-law dependence on the overdrive voltage:  
  Our threshold voltage remains constant for a given MOSFET

is a constant for a given MOSFET

**Why is NMOS advantageous**

* Electron mobility is 2-4 times greater than hole mobility .

## P-Channel MOSFET

## N - Channel MOSFETS

**Triode Mode when**

**Saturation mode when**

**MOSFETS** amplify in Saturation Mode

**BJT** Amplify in Active Mode

## Voltage Gain

In saturation (Q):

**Gain:**

A max is typically ~10.

## Small Signal Analysis

DC voltages (VGS and VDD) set the bias point (Q) and AC signal vgs is amplified.

### Example

Consider the amplifier shown,

**Transistor:**  
  
  
 = 10

a) For Vgs = 0 (hence Vds = 0) find Vov I, VDS and Av  
b) DC bias point: what mode?

Saturation with VOV = 0.2V and ID = 0.08mA, VDS = 0.4V  
Av = -14

c) maximum symmetrical swing allowed at the drain? Hence find maximum allowable amplitude of sinusoidal   
d) What is the criterion for our gain calculation to be applicable

hence allowed swing at is 0.2V Swing.

## Small Signal Approximation - Saturation Region

Similar technique for linearization as for the exponential diode and the BJT. No need for Taylor Series.

If

## Examples

**Example 1 - nMOSFET**

**Example 2 - nMOSFET**

**Example 3**

**Example 4**

### MOSFET Long Answer

### Example

# Power Semiconductors

## Power Diodes

Power diodes differ from signal diodes in their construction, instead of a simple PN junction, there are extra layers with different doping

### Parameters

* Diodes have several key parameters, some of which are constant and some which vary with condition
* **Voltage Rating** - Max instantaneous voltage the device can block in the off state
* **Current Rating** - Max instantaneous average or RMS current that it can conduct in the ON state
* **Switching Speed** - Transition speed from on to off
* **On State Voltage** - Voltage dropped across the device when it is conducting

### Switching State

* Their on and off states controlled by the power circuit
* Diode turn-off is not instant, a sudden change in polarity will not immediately stop current
* There is an **additional charge** that needs to be supplied to complete turn-off. The diode conducts a **negative current for duration** .
* Known as **Reverse Recovery**
* Power diodes are classified based on their reverse recovery characteristics. General/Fast-Recovery/Shottky

### Diode Types

* **General Purpose Diode**
  + Relatively high (~25 microseconds)
  + Good for low frequency applications up to ~1kHz
  + Typical current ratings 1-1000A, voltage ratings 50V-5kV
* **Fast Recovery Diodes**
  + Relatively low (<5 microseconds)
  + Good for power conversion systems
  + Typical current ratings 1-1000A, voltage ratings 50V - 3kV
* **Schottky Diodes**
  + These have a metal/semiconductor junction rather than PN
  + Very fast switching (low in the nanoseconds)
  + Typical current ratings 1-300A voltage ratings ~100V

### Applications

* Freewheeling diodes/clamp diodes/snubber diodes
* AC/DC conversion/rectification - Changing between alternating and direct current
* Battery Charging

## Power BJTs

* BJTs can be used for amplifiers and switches
* For **power switching** applications the **cut-off and saturation regions** are used
* Similar to power diodes **high power rated BJTs have an additional n-region**
* Generally superseded by other tech. Cheaper than MOSFETs
* Used in output stages of audio amps, touch sensitive switches, computer controlled relays, low power AC/DC supplies

### Circuit Configs

* Usually used in the common emitter configuration
* To handle higher switching currents a darlington pair/triple darlington config can be used.
* This config can be generally treated just like a single transistor but with

Voltage drop also increases

## Power MOSFETs

* Power MOSFETs are the most common power semiconductor
* Power MOSFETs are designed to handle higher power levels
* High switching speed
* Good low voltage efficiency
* Often low gain
* Commonly used for "low voltage" switching (<200V)

### Structure

* Made using silicon and fabricated as a vertical diffused MOS structure
* Source is above the drain, current flow is primarily vertical
* Vertical structure means that the *voltage rating depends on the doping and thickness of the N+ Layers* whilst the **current depends on the channel width.**
* This design allows for **higher currents** and *power ratings* than the traditional lateral MOSFET.
* Typically up to 200V
* Current to ~100A
* Frequencies in excess of 100kHz
* Used for high power switching
* Applications include
  + Power supplies DC-DC converters, low voltage motor controllers, vehicle electronics

## IGBT (Insulated Gate Bipolar Transistor)

* Combine the ease of control of a MOSFET even at higher voltages (>200V)
* Circuit symbol is similar to BJT but with an extra line

### Parameters

* Voltage ratings up to ~5kV and current ratings up to ~2000A
* Most commercial designs do not block reverse polarity voltages
* Typically used for converters over a wide power range (1kW up to >1MW)
* Used in motor drive circuits, UPS, induction cooktops

## Thyristors

* Four layer semiconductor devices with alternating doping region e.g. PNPN
* Essentially a semi-controllable diode
* Three electrodes, anode cathode and gate.
* Various types of thyristors are available. Most common is the **Silicon Controlled Rectifier** (SCR)
* Work as a **bistable switch**, **conducting when there is a current trigger at the gate**, *they keep conducting until a reverse bias is applied.*
* *Only a short pulse is needed to turn diode on*
* SCR is the most common type of Thyristor
* Like a modified diode, SCRs are unidirectional they only conduct current in one direction

**Modes of Operations**

* *Forward Blocking Mode* - Anode has +ve voltage and cathode has -ve, gate held at a zero potential, only a small leakage current flows from A to C.
* *Forward Conduction Mode* - As above but potential between anode and cathode is now increased beyond breakdown, **OR** a positive pulse is sent to the gate that is now in the ON state.
* *Reverse Blocking Mode* - Anode has **-ve voltage and cathode has +ve behaves** like t**wo diodes in series**, only a small leakage current flows.  
  SCRs are typically used in medium-high voltage control (power regulator light dimmer)

### Gate Turn Off Thyristors (GTO)

* Provides additional control
* GTO the gate can be used to turn off the device (unlike with a regular SCR)
* Requires a negative signal at the gate to turn-off
* Has the drawback of long switch off times, use for low switching speed (up to 1kHz) - can use a Snubber Circuit to reduce turn off time.
* Applications include high speed motor drives and high power inverters

### MOS Controlled Thyristors (MCT)

* Essentially consisting of a thyristor with two MOSFETs built into the gate
* These MOSFETs are used to turn the gate on and off
* A negative pulse (relative to anode) turns the device on
* MCTs offer
* Low forward conduction loss
* Fast switching
* High input impedance at gate

# AC Circuits and Power Factor Correction

Kirchoff Current and Voltage Laws (KCL and KVL) - quick example

**KCL:**   
**KVL:**

One per branch

**Instantaneous Power:** Measured in Watts  
**Energy:** Measured in Joules  
**Average Power:** Measured in Watts

## AC Circuits in the Time Domain

**RMS Value**

Angular Frequency  
 Angle of @   
 Angle of @   
 Angle difference between v and i

i(t) lags v(t)  
 i(t) lags v(t)

In this example:  
 , ,

lags , after

### Single Phase Instantaneous Power

**Where the first half is pr(t) and px(t)**

Current Component in phase with   
 Current Component 90 degrees out of phase from   
,

* Instantaneous power pulsates @ .
* Can be positive or negative

## Active Power, Reactive Power, Power Factor

**Definitions:**

* **REAL/ACTIVE Power** (W)
* **Reactive Power** (VAr)
* phase angle difference between **v** and **i** respectively

### Example: Purely Resistive Load

### Example: Purely Inductive Load

### Example: Purely Capacitive Load

![](data:image/png;base64;base64,)

Sine and Cosine

Where

## Phasors

**Assumption**

* Constant frequency, i.e. is fixed

![](data:image/png;base64;base64,)

Note

**Reminder**: is .

is

Cosinusoidal quantity quantity x(t) (Voltage, current, whatever) characterised by:

* Maximum value
* Phase angle

Where: = RMS, effective value.

Using Euler's Identity

![](data:image/png;base64;base64,)

Important

We use . This may be different to other units, and is a convention in electrical engineering.

### Phasor Operations

**Polar Form**  
Coordinates () Useful for multiplication and division

**Rectangular Form**  
Coordinates (Re, Im  
Useful for addition and subtraction:

Given:

### Phasor Operations

**Polar Form**  
Coordinates () Useful for multiplication and division

**Rectangular Form**  
Coordinates (Re, Im  
Useful for addition and subtraction:

Given:   
***Derivation in the time domain is multiplication by jw in the frequency domain***

***Integrationin the time domain is division by jw in the frequency domain***

**Resistor** - Has no effect on phase (In phase)  
**Inductor** - Current *Lags* Voltage  
**Capacitor** - Current *Leads* Voltage

**Impedance**:

**Admittance**:

* Ohms law can be written in the RMS phasor domain for any passive circuit element as:
* Series and parallel combination of passive elements imply that the impedance has a real and imaginary component.

and

#### Example

Using RMS phasor approach, determine the time domain expression for the current in a circuit by the differential equation:

### Series and Parallel

## Complex Power

Power definition in the time-domain **instantaneous power**

Power definition in the phasor domain **complex power**

**Where:**  
 and are the voltage and current rms phasors,  
and is the complex conjugate of , i.e. ()

**Resistor:**

**Inductor:**

**Capacitor:**

For a general **(Passive) RLC circuit** with the load convention: **P > 0,** *Q > 0 for inductive loads* (Q is absorbed) *Q < 0 for capacitive loads* (Q is sourced)

**Power Factor**  
Sometimes is used instead of .

## Power Factor Correction

We can correct the power factor by altering the load as seen by the voltage source .

For (a). Without capacitor, angle between total load current and voltage is   
For (b). With capacitor, angle between total load current and voltage is

**Case A**

**Case B**

![](data:image/png;base64;base64,)

Important

### Example

### Capacitive Load Example

If the load is capacitive, and we want to increase power factor, then a shunt inductor can be used.

# 3 Phase AC Power

## Balanced 3 Phase Circuits

**Balanced 3 phase circuit has**

* 3 voltage sources with equal magnitude, but with an angle shift of
* Equal loads on each phase
* Equal impedance on the lines connecting the generators to the loads

## Line to Neutral and Line to Line Voltages

| Balanced line to neutral voltages | Balanced line to line voltages |
| --- | --- |
|  |  |
|  |  |
|  |  |

### Balanced Line to Neutral Voltages

**Balanced Conditions:**

**KVL:**  
Sum of line to line voltage is **always zero**.

### Balanced Line Currents

**KCL** node N :   
**Balanced conditions**:

### Balanced loads

**Load** currents under balanced conditions:  
**Line** current equations:  
*KCL (A)*:   
*KCL (B)*:   
*KCL (C)*:

## - Y Conversion for Balanced Loads

Posing equality of **line** currents for the circuit on the left and on the right:

is the same in both circuits if:

## Power in Balanced circuits

### Instantaneous Power

Instantaneous power for phase a:

In the same way, power for **phase b and c can be calculated**

The instantaneous power is:

is instant. **POWER IS NOT A FUNCTION OF TIME**

### Complex Power

#### Complex power in balanced Connected Circuits

**Complex Power for Phase A:**

**Active Power:**

**Reactive Power:**

**Apparent Power:**

# Magnetic Circuits

Flux density (magnetic induction) or B-Field

**Where:**  
H = Field intensity  
 = Magnetic Permeability (material)  
 = Air Permeability  
 = Relative Permeability

## Coil Around a Core

Induced by a change in flux   
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**One turn**

**N Turns**

We know that where L is constant.

![](data:image/png;base64;base64,)

Note

Inductance Definition:

Magnetic energy is stored in an inductor in (Joules)

Is energy stored due to magnetic field inside a meterial ( is a material-dependent constant)

Magnetic energt density,

𝓁𝓁

The inductance can be determined using magnetic circuit relations:

# Transformer

**Ideal Transformer**

*Faraday's Law:*$$  
\delta(t) = \begin{cases}  
v*{1}=n*{1} \frac{\Delta\Phi}{\Delta t} \  
v*{2}=n*{2} \frac{\Delta \Phi}{\Delta t}\  
\end{cases}

![](Attachments/Pasted%20image%2020231026171624.png) $$\frac{\Delta \Phi}{\Delta t}=\frac{v\_{1}}{n\_{1}}=\frac{v\_{2}}{n\_{2}}\rightarrow \frac{v\_{1}}{v\_{2}}=\frac{n\_{1}}{n\_{2}}$$ \*\*Actual Transformer:\*\* $\mathscr{R}>0$ Magnetizing Inductance $L\_{m}$ $$\mathscr{R}\Phi=n\_{1}i\_{1}+n\_{2}i\_{2}$$ $$\Phi= \frac{n\_{1}i\_{1}+n\_{2}i\_{2}}{\mathscr{R}}$$ ![](Attachments/Pasted%20image%2020231026172014.png) \*\*Faraday's Law\*\* $$v\_{1}=n\_{1} \frac{d \Phi}{d t} = \frac{n\_{1}^{2}}{\mathscr{R}} \frac{d}{d t} \left( i\_{1} + \frac{n\_{2}}{n\_{1}} i\_{2}\right)$$ $$v\_{1}=\frac{n\_{1}^{2}}{\mathscr{R}} \frac{d}{dt} \left( i\_{1} + \frac{n\_{2}}{n\_{1}}i\_{2} \right)$$ For DC input Voltage $v\_{1}=V\_{dc}=$ constant $\frac{d}{dt}\left( i\_{1}+ \frac{n\_{2}}{n\_{1}}i\_{2} \right)$ = Constant $i\_{M}$ grows at constant rate, i.e. the primary sees a short-circuit Secondary Disconnected $(i\_{2}=0)$ ## Transformers in AC Circuits \*\*Ideal Transformer\*\* $$\bar{V}\_{1} = n\_{1} \frac{d \bar{\Phi}}{dt} = j \omega n\_{1} \bar{\Phi}$$ $$\bar{V}\_{2}=n\_{2} \frac{d \bar{\Phi}}{dt} = j \omega n\_{2} \bar{\Phi}$$ $$\frac{\bar{V}\_{1}}{\bar{V}\_{2}} = \frac{n\_{1}}{n\_{2}} = a\_{t}$$ $a\_{t}$ is the turns ratio \*\*- Complex power entering winding 1 equals the complex power exiting winding 2.\*\* $\bar{S}\_{1} = \bar{S}\_{2}$ $$\bar{V}\_{1}\bar{I}\_{1}^\* = \bar{V}\_{2}\bar{I}\_{2}^\*$$ $$\bar{S}\_{1} = \bar{V}\_{1}\bar{I}\_{1} = a\_{t} \bar{V}\_{2} \frac{\bar{I}\_{2}}{a\_{t}}=\bar{S}\_{2}$$ $$V = n \frac{d \Phi}{dt}\rightarrow \frac{\bar{V\_{1}}}{\bar{V}\_{2}} = \frac{n\_{1}}{n\_{2}}=a\_{t}$$ $$n\_{1}\bar{I}\_{1}=n\_{2}\bar{I}\_{2}\rightarrow \frac{\bar{I}\_{1}}{\bar{I}\_{2}} = \frac{n\_{2}}{n\_{1}} = \frac{1}{a\_{t}}$$ \*Impedance $Z\_{2}=\frac{\bar{V}\_{2}}{\bar{I}\_{2}}$ seen from winding 1 $$Z\_{2}' = \frac{\bar{V}\_{1}}{\bar{I}\_{1}} = \frac{a\_{t}\bar{V}\_{2}}{\frac{\bar{I}\_{2}}{a\_{\_{t}}}} = a\_{t}^{2}Z\_{2}$$ ## Practical Transformers \*Magnetic core reluctance $\mathscr{R}>0$\* ![](Attachments/Pasted%20image%2020231029143042.png) $$\bar{V} = n \frac{d \bar{\Phi}}{dt} \rightarrow \frac{\bar{V}\_{1}}{\bar{V}\_{2}} = \frac{n\_{1}}{n\_{2}} = a\_{t}$$ $$n\_{1} \bar{I}^{'}\_{1} = n\_{2} \bar{I\_{2}} \rightarrow \frac{\bar{I}\_{1}}{\bar{I}\_{2}} = \frac{n\_{2}}{n\_{1}} = \frac{1}{a\_{t}}$$ \*With core reluctance>0 and windings leak some flux outside core.\* - Flux is leaked outside the core by each winding $\Phi\_{l\_{1}}$, ${\Phi\_{l\_{2}}}$ - Leaked flux is modelled by leakage inductance $L\_{l1}$, $L\_{l\_{2}}$ - R1 and R2 represent winding's copper losses ![](Attachments/Pasted%20image%2020231029143738.png) \*Core reluctance > 0, windings leak flux, and core dissipates active power (Hysteresis)\* - Due to Hysteresis of the B-H curve, energy is dissipated inside the core to create magnetizing flux - Energy loss in the magnetization process is represented by an additional shunt resistance $R\_{M}$ in parallel with $L\_{m}$ ![](Attachments/Pasted%20image%2020231029144118.png) ## Example ![](Attachments/Pasted%20image%2020231029144421.png) ![](Attachments/Pasted%20image%2020231029144435.png) # Renewable and Solar ### Load Convention \*\*Without Light\*\* $$i = I\_{0}(e^{x}-1)$$ note that $x \propto v$ \*\*When light is shone on the "diode"\*\* $$i = I\_{0}(e^{x}-1) - \text{const}$$ $\text{const} = I\_{sc}$ ## Anti-Parallel Diode Model ![](Attachments/Pasted%20image%2020231026192503.png) $$I = I\_{sc}-I\_{d}$$ $$I = I\_{sc} - I\_{0} (e^{\frac{qV}{kT}}-1)$$ $q = 1.602 \* 10 ^{-19}C$ : Electron Charge $k= 1.38\*10^{-23}$ J/K: Boltzmann Constant $V\_{T} = \frac{kT}{q}$ V: Thermal Voltage $T$: Temperature in \*\*K\*\* at 25C $T = 273.15 +25 = 298.15K$ $I = I\_{sc} - I\_{0}(e^{38.9V}-1)$ For $I = 0, V=V\_{oc}$ $$V\_{oc} = \frac{kT}{q}\ln\left( \frac{I\_{sc}}{I\_{0}}+1 \right)$$ At 25C for $I=0$ $0=I\_{sc}-I\_{0}(e^{38.9V\_{oc}}-1)$ $V\_{oc} = 0.0257 \ln\left( \frac{I\_{sc}}{I\_{0}}+1 \right)$ - Isc varies proportionally with irradiation - Voc varies logarithmically with irradiation (does not change much) ![](Attachments/Pasted%20image%2020231026195224.png) The PV module delivers the maximum power only at one operating point (MPP) $V = V\_{R}, I=I\_{R}$ ### Inclusion of $R\_{s}$ and $R\_{p}$: PV Module i-v curve ![](Attachments/Pasted%20image%2020231026205101.png) $$=I\_{sc} - I\_{0} \left( e^{\frac{q(V+R\_{s}I)}{kT}}-1 \right)-\frac{V\_{d}}{R\_{p}}$$ Where the $I\_{0}$ portion is $I\_{d}$, and the fraction at the end is $I\_{p}$ $V = V\_{d}-R\_{s}I$ $V\_{\text{module}}=n(V\_{d}-R\_{s}I)$ $$=I\_{sc} - I\_{0} \left( e^{\frac{q(V+R\_{s}I)}{kT}}-1 \right)-\frac{V+R\_{s}I}{R\_{p}}$$ ### Impact of Shading (using non-ideal PV cell circuit) ![](Attachments/Pasted%20image%2020231026210422.png) \*\*Full sun\*\* - All cells produce I - PV module voltage V \*\*One cell is shaded\*\* - Fully shaded cell produces no current $I\_{sc} = 0$ - Shaded cell diode is reverse biased - PV module voltage V < Vsh $V-V\_{SH} = \Delta V$: PV Module voltage reduction due to shading on one cell - n-1 cells in full sun: Generator. $V\_{n-1} = V (\frac{n-1}{n})$ - n-th shaded cell: $R\_{p} + R\_{s}$ load resistor (Rp>>Rs) - Shaded cell absorbs power $R\_{p}I^{2}$ - Shaded cells heat up because of power absorption $$\Delta V = V - V\_{SH} = V - V (\frac{n-1}{n}) + (R\_{p} + R\_{s})I = V/n + (R\_{p} + R\_{s}) = I$$ #### Bypass Diodes \*Shading effect mitigation:\* ![](Attachments/Pasted%20image%2020231026210923.png) \*\*Without Bypass Diode:\*\* - Current flows through $R\_{p}+R\_{s}$ - Shaded cell causes large voltage reduction - Shaded cell voltage $(R\_{s}+R\_{p})I$ is large \*\*WITH Bypass Diode:\*\* - Current flow is diverted through BP -diode - Shaded cell causes small voltage reduction - Shaded cell voltage is small (typically 0.5 - 1V) ## Peak Power Operation - We want to operate the array at the max power point (MPP) - The maximum power is extracted by the PV module as long as it operates at $V\_{mpp}$ $$\eta = \frac{V\_{mpp}\*I\_{mpp}[W]}{1000\left[ \frac{W}{m^{2}} \right] \* \text{Area}[m^{2}]}$$ # Batteries - \*\*Energy Density:\*\* Energy stored per unit of mass volume, in $Wh/kg$ or $Wh/m^3$ - \*\*Power Density:\*\* Max rate of energy discharge per unit mass or volume in $W/kg$ or $W/m^3$ - \*\*Capacity:\*\* Amount of charge deliverable by a battery in a discharge cycle in $Ah$ - \*\*Life:\*\* number of discharge/recharge cycles at the rated capacity ## Energy Density and Power Density \*\*Lead Acid\*\* $Q = 80Ah$ $V = 12V$ $m = 31kg$ $v=10dm^{3}$ $R\_{int}=37.5 m \Omega$ ![](Attachments/Pasted%20image%2020231031143628.png) $W = 12V 80Ah = 960Wh$ $\frac{W}{m} = \frac{960Wh}{31kg} = 31 \frac{Wh}{kg}$ \*\*Specific Energy (energy density)\*\* $P\_{max} = \frac{\left( \frac{V}{2} \right)^{2}}{R\_{load}} \approx 1kW$ \*\*theoretical max power when $R\_{load} = R\_{int}$\*\* $\frac{P\_{max}}{m} = 32 \frac{W}{kg}$ \*\*Specific power (power density)\*\* ![](Attachments/Pasted%20image%2020231031144307.png) ## Equivalent Circuit ![](Attachments/Pasted%20image%2020231031144834.png) A 200Ah battery - Delivers 10A for 20h - Delivers 50A for 4h ### Discharge Rate - Rate at which charge is taken out of the battery - Example: discharge rate for 200Ah battery - 1h is 1C = 200Ah (1C --> 1 \* 200A = 200A in 1 hr) - 10h is 0.1C = 20A (0.1C --> 0.1 \* 200A = 20A in 10hrs) - 0.5h is 2C = 400A (2C --> 2 \* 200A = 400A in 0.5h) ![](Attachments/Pasted%20image%2020231031151346.png) - Min voltage depends on discharge rate - Higher discharge rates imply lower min voltage - Higher discharge rates imply lower discharge times ### Typical charging profile Constant current - constant voltage charging profile (CC-CV) ![](Attachments/Pasted%20image%2020231031152100.png)